Saturday, June 26 AM 9:00-12:00, Room 140

Session 2 (room 140): **History of Math**

- Jean-Louis Hudry, “Hilbert’s Implicit Definitions and their Syntactic Consistency”
- K.A. Brading and T.A. Ryckman, “Hilbert’s Second Communication on the Foundations of Physics”
- Martin Neumann, “Measuring the Uncertain: A Theory of Single Case Probabilities”
- William Goodwin, “Syntheticity and Generality in Kant’s Philosophy of Geometry”

“Hilbert’s Implicit Definitions and their Syntactic Consistency”

Jean-Louis Hudry

Frege defends a traditional conception of axioms, defined as unproved, true propositions. The truth of an axiom is not derived from the axiom system itself, but from the explicit definition of an object falling under the concept. Frege is then confronted to the following questions. What is a mathematical object? Is Julius Caesar a number?

We may avoid such questions by supposing that mathematical concepts are not of the same kind as the intuitive concepts of ordinary language. In this sense, Hilbert’s ‘Grundlagen der Geometrie’ (1899) and ‘Über den Zahlbegriff’ (1900) define two possible methods. By dealing with geometry, a genetic method postulates some geometric elements (like in Euclid’s *Elements*) whose explicit definitions resort to intuition. Then the truth of axioms rests on the intuitively and explicitly true definitions of geometric objects, such that a point is a partless entity or a line a breadthless segment. Yet, Hilbert develops another method called axiomatic method and based on the syntactic consistency of an axiom system. This consistency does not depend on a logical truth condition but is an axiomatic truth implying the production of a model (i.e. an interpretation) such that the axioms of the system do not contradict each other. An axiom system has no semantic content since it pertains to neither the explicit definitions of mathematical concepts nor existing mathematical objects falling under such concepts. In other words, the concepts of the axiom system constitute explicitly undefined primitives, and the only way to apprehend them is through their implicit definitions provided by the axioms. Then syntactic consistency requires that each implicit definition does not contradict the rest of the implicit definitions within the axiom system. If we produce a model in which an axiom contradicts the rest of the axioms, then this axiom is said to be independent. For instance, the system of axioms $S\&\neg A$ is consistent if and only if A is an independent axiom.

Furthermore, a model is complete if it is maximal and isomorphic to another model (see Hilbert’s introduction of the Axiom of Completeness in the French translation from 1902). Two models are isomorphic if there is a one-to-one correspondence between their elements that preserves all relations, such that the model of Euclidean points is isomorphic to the model of real numbers. In this sense, Hilbert’s abstract model of

Euclidean geometry cannot be confused with Frege's concrete model of Euclid's geometry. The former is based on the axioms themselves, called implicit definitions, while the latter requires intuitive, explicit definitions on which the axioms depend. Consequently, Hilbert's axiomatic system of implicit definitions is incompatible with Frege's logical system of explicit definitions, since the syntactic consistency of any abstract model does not match the semantic consistency of any concrete model.

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"Hilbert's Second Communication on the Foundations of Physics"

K.A. Brading and T.A. Ryckman

In November and December 1915, Hilbert presented two communications to the Göttingen Academy of Sciences under the common title "The Foundations of Physics". Versions of each eventually appeared in the Nachrichten of the Academy. The first has attained widespread recognition; bearing 20 November as the date of its submission to the Nachrichten, five days before Einstein presented the final form of his generally covariant gravitational field equations to the Berlin Academy, the printed version of 31 March 1916 contains essentially the same gravitational field equations as those of Einstein. However, these equations do not explicitly occur in the recently-discovered proofs of Hilbert's first communication, dated 6 December 1915. In contrast, Einstein's presentation had already appeared in print on 2 December. The attention to dates is not pedantry for it raises a number of questions, including whether, as several scholars have recently alleged or insinuated, the proofs provide evidence of an attempt at "nostrification" by Hilbert of Einstein's final results. We disagree with this suggestion, and with the associated interpretations of Hilbert's project. By regarding Hilbert's two contributions as part of a research program within the overarching framework of "the axiomatic method" (as Hilbert expressly stated was the case), we see that their primary concern was not a "race" with Einstein for the generally covariant gravitational field equations, but rather the collocation of Einstein's theory of gravitation and Mie's theory of matter (these being two theories that, in 1915, might well have been considered fundamental) under a single generally invariant action function. This was the central achievement of the first contribution, and one that is preserved between the proofs and the published version. The second contribution, which did not reach the printer until January 1917, has been largely either disregarded or misinterpreted. The discovery of the first contribution proofs has made it possible to see the thematic linkage between the material that Hilbert cut from the first contribution and the content of the 1917 second contribution. This lies in the apparent tension between general invariance and causality (in the precise sense of Cauchy determination), pinpointed in theorem 1 of the first contribution. This is not the same problem as that found in Einstein's "hole argument" – something which, we argue, never confused Hilbert. The solution attempted in the proofs, involving 4 non-covariant energy equations, failed; this, together with all discussion of the problem, is the material cut from the proofs. The second contribution shows that the conflict can be resolved by using special coordinate systems on the initial data spatial 3-surface. In this way, Hilbert initiated work on the initial value problem in general relativity. Furthermore he stated that the new field physics was a “pseudo-geometry”, inconsistent with any remnants of Euclidean action-at-a-distance geometry. This led to a careful investigation of where, and in what circumstances, the “pseudo-Euclidean” assumptions of Minkowski space-time enter into the new physics. In particular, Hilbert showed that the Schwarzschild exact solution for the solar gravitational field could be obtained without positing Minkowski values of the metric tensor “at infinity”.

“Measuring the Uncertain: A Theory of Single Case Probabilities”

Martin Neumann

This talk is devoted to a theory of single case probabilities of the late 19th century, the so-called theory of ‘Spielrהume’. It was developed in 1886 by the German physiologist Johannes v. Kries. Nowadays theories of single case probabilities usually refer to propensity theories in the line of Karl Popper. However, this theory is settled in a significantly different philosophical framework. It will be shown that this is a consequence of 19th centuries research questions in the fields of both psycho-physical measurement and Laplacian probability theory.

The main theoretical terminus of this theory is called ‘Spielraum’ (i.e. range or scope). Thus, in a first step this term will be introduced. To illustrate it, a simple game of chance which will be analysed, called ‘Sto□spiel’.

The next step is concerned with the measurement of a ‘Spielraum’. v. Kries follows the tradition of Laplacian probability theory. Within this context, a highly controversial discussion was the question of what are cases of equal possibility.
v. Kries wanted to solve this problem by measure theoretical considerations. They developed out of problems of psychological measurement, which turned his attention to probability theory. Following v. Kries, the size of two ‘Spielrהume’ can be compared if they fulfil the condition of so called indifference, originality, and comparability. These conditions will be examined in detail.

Finally, it will be identified what actually is measured by the comparison of two of ‘Spielrהume’.

v. Kries distinguishes between what he calls nomological and ontological aspects of the description of nature. Natural laws are nomological aspects, while the circumstances of a single event are ontological aspects. The concept of ‘Spielrהume’ is relevant for the latter. Therefore probability theory tells nothing about nomological determinism or indeterminism. Instead ‘Spielrהume’ are intended as a measure for ontological vagueness. Hence, contrary to modern accounts on propensities, which usually refer to an indeterministic world view, ‘Spielrהume’ are intended to reconcile probability theory with determinism.

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"Syntheticity and Generality in Kant's Philosophy of Geometry"

William Goodwin

In this paper, I explore the relationship between Kant's claim that the truths of geometry are synthetic and his explanation of the generality of these truths. I begin by characterizing what is commonly known as Berkeley's Generality Problem and by tracing the origin of this problem, for both Berkeley and Kant, to their attempts to provide an alternative to a common, early modern conception of the nature of mathematical reasoning. Whereas Berkeley is forced to deal with the Generality Problem because he rejects the coherence of abstract ideas, Kant encounters this problem because he finds that appeals to the intensions of concepts are not sufficient to explain our access to the necessary truths of geometry. Kant's positive doctrine of the synthetic a priori character of geometrical judgments is an attempt to provide such an explanation; however, this doctrine is also the source of the need for a solution to Berkeley's Generality Problem.

Both Berkeley and Kant propose solutions to Berkeley's Generality Problem that depend upon developing a sense in which an individual is representative of the extension of a concept. In the second half of the talk, I explore the prospects for explaining the generality of Euclidean proofs in terms of such representative individuals. By focusing on certain Euclidean theorems that require proofs by cases, I argue that ultimately this sort of solution is undermined by the dependence of Euclidean proofs on 'unrepresentative' topological features recognized on the basis of the accompanying diagram. As a result, the dependence of the Euclidean practice on its diagrams can not only be seen to be what makes Kant's account of the synthetic a priori character of geometrical theorems compelling, but also what undermines the explanation of the generality of geometrical theorems necessitated by this doctrine. More specifically, the role of diagrams in facilitating geometrical reasoning not only explains why Euclidean reasoning cannot be carried our at a purely conceptual level and instead must appeal to representations of individuals, but also why it is not possible (in general) to understand these individuals as representative of the concepts under which they fall.

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