

Friday, June 25 AM 9:00-12:00, Room 100

Session 2 (room 100): **Philosophy of Mathematics**

Audrey Yap, “Congruences and Quadratic Reciprocity”

Dirk Schlimm, “Axiomatics and Discovery in 20th Century Philosophy of Science and Mathematics”

Michel Serfati, “Some Elements for a Philosophy of Contemporary Mathematics”

Yvon Gauthier, “Hermann Weyl on Riemannian Geometry”

“Congruences and Quadratic Reciprocity”

Audrey Yap

The law of quadratic reciprocity states that if p and q are distinct odd primes, then the two congruences $x^2 \equiv q \pmod{p}$ and $x^2 \equiv p \pmod{q}$ are either both solvable or both unsolvable, unless p and q are both of the form $4k + 3$, in which case one congruence is solvable and the other is not. Gauss is credited with the first proof of this law, given in *Disquisitiones Arithmeticae*.

This work is also where he introduces his new theory of congruences, which is used in the proof of the law. In the discussion following his proof, he remarks on the fact that no one had thus far presented it in so simple a form. Furthermore, he finds it remarkable that Euler knew other propositions which depend on it and should have led to its discovery, yet did not prove the law himself. Legendre also worked on the problem, yet had an incomplete proof.

Now, everything that can be stated in terms of congruences can also be stated in terms of divisibility. Yet in a letter to Schumacher, Gauss wrote that new calculi can play a significant role in problem-solving, though in general, one cannot attain anything by them that could not also have been attained without them. He goes so far as to say that they allow mechanical solutions in cases “where genius itself becomes impotent.” We can look at the theory of congruences as being such a new calculus.

On the face of it, the theory of congruences looks like a particularly innocuous example of a new calculus, but we can see Gauss as crediting it with his proof of the law of quadratic reciprocity. In this paper, I will look at the relationship between Gauss' new theory of congruences and the proof he discovered, as part of a more general question about the epistemological relevance of representation in mathematics.

“Axiomatics and discovery in 20th century philosophy of science and mathematics”

Dirk Schlimm

Ever since its inception the axiomatic presentation of geometry in Euclid's *Elements* has been heralded as a prime example of a great intellectual achievement and as

an ideal upon which innumerable scientist, mathematicians and natural philosophers alike, modeled their own work. To be sure, geometry did not end with Euclid. Rather, his axiomatization has been the source of numerous and fruitful developments. The invention of modern logic together with the desire for rigorous proofs without appeal to intuition have led to an increased attention to axiomatics in the late 19th century. This development culminated in Hilbert's vision of the “formal” axiomatization of all mathematics and natural sciences, which provided a cornerstone of analytic philosophy in the early 20th century. Today, axiomatics belongs to the toolbox of every scientific enquirer.

With this development as the background, I argue that the contribution of axiomatics for the advancement of science and mathematics has not been properly acknowledged in the philosophical literature by presenting an overview of how axiomatics was viewed in 20th century philosophy of science (e.g., Carnap, Hempel, Hanson, Hesse, Kuhn, Lakatos, van Fraassen) and philosophy of mathematics (e.g., the Benacerraf/Putnam collection, Lakatos, Polya, Kitcher). I show that in the context of scientific methodology, in particular regarding its use in the context of discovery, axiomatics has received only scant attention. This is a rather surprising result, since axiomatics has been employed extensively in mathematics, science, and also by the philosophers themselves. I discuss possible reasons for and shortcomings of this development in philosophy.

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“Some Elements for a Philosophy of Contemporary Mathematics”

Michel Serfati

The word “contemporary”, when applied to mathematics, must be understood here opposed to “modern”. By modern, I mean early XXth mathematics, (roughly) up to Emmy Noether and Hilbert, the most influential treatise of which doubtlessly being Van der Waerden’s *Modern Algebra*— this point is extensively studied by Leo Corry. On the contrary, by contemporary mathematical ideas, I will mean here some of these important mathematical ideas which, on the one hand, arose after World War II, and on the other hand principally, are to-day still in use and fruitful. Along these lines, some Marshall Stone’s results in the 1930’s — among which one stresses a well-known main theorem — turned out to be a decisive turning point between the two periods, an early breakthrough from which arose completely new methods and mathematical conceptions, mainly between algebra and topology. Thus, these “elements for a philosophy” will appear, at the end of the present conference essentially devoted to Stone, as three main figures of thought, first “ideal thought”, that is contemporary spectral methods, following the primitive correspondence stated by Stone between boolean rings and so-called “Stone spaces”. A second, though connected to the first figure, is “duality”, as it appeared firstly in Stone-Gelfand’s duality— it gave birth to the contemporary studies of C*-algebras. Actually, the concept of duality filled a decisive permanent place in early Mac Lane’s works. Finally, as quoted by Mac Lane himself, the influence of Stone’s early

very specific conceptions was most important for the constitution and the development of some of the main concepts of category theory, such as adjoint functors : this organized a third figure, “categorical thought”.

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“Hermann Weyl on Riemannian Geometry” **Yvon Gauthier**

On various occasions, Weyl has discussed Riemannian geometry not only from a mathematical point of view, but also from his own original philosophical perspective. In his comments on Riemann’s «Ueber die Hypothesen welche der Geometrie zugrunde liegen » (see 3), Weyl stresses the fact that “pure infinitesimal geometry” is the application of the Pythagorean theorem in the infinitely small (see also 4). He also points out the local character of a physical geometry in which the inertial and the metrical structure of the world are intimately connected. But it is in his major work *Raum, Zeit, Materie* (1918, see 5) that Weyl puts to use his own idea of the affine infinitesimal geometry of a four-dimensional space.

I want to put the emphasis on Weyl’s own achievement while taking the full measure of the mathematical physics inaugurated by Riemann. In this assessment, one should not forget the « Pariser Arbeit » where Riemann shows concern for the application of quadratic forms to the isothermy problem (see 2).

It is to his interests in foundational problems that Weyl has devoted his main intellectual efforts and I wish to stress the constructivist inspiration from *Das Kontinuum* to the late work on the algebraic theory of numbers. His attempt at a unified field theory must be replaced in the same spirit of the theoretical construction of the world, a theme which is recurrent in his more philosophical writings. In that respect, one could say that Riemann and Weyl shared a common, if somewhat imprecise, Kantian background (see 1).